Lecture 11 - Work, Power and Energy II

Introduction

We now know that mechanical work is defined as,

$$W = \sum_i \vec{F}_i \cdot \Delta\vec{r}_i \tag{1}$$

If the force is a constant, which is often the case in the problems that we do, this reduces to,

$$W = \vec{F} \cdot \Delta\vec{r} \qquad \text{If force is constant} \tag{2}$$

Note that this is the work done by the force $\vec{F}$ in moving a body a distance $\Delta\vec{r}$. Note that the work does not tell us what happens to the velocity of the body. It may accelerate if the force $\vec{F}$ is not balanced by an opposing force so in this case the force is converted to kinetic energy. If the force $\vec{F}$ is perfectly balanced, then the work may be converted to potential energy, for example if we lift a body up. Finally the work may be dissipated, for example if the force $\vec{F}$ is perfectly balanced by a frictional force $\mu_k N$. In general, work is converted to a combination of these forms of energy. If there is no friction, the system is called conservative as none of the energy is dissipated. If there is friction or drag, then the system is non-conservative and some of the work is "lost" due to dissipation.

Another type of force - A spring

A spring sounds pretty boring and low tech. However it turns out that a spring is a pretty good model for many really interesting things. For example the deformations of proteins can be modelled using springs as can the vibrations of atoms in crystals. Consider placing a spring along the axis. When we apply a force of magnitude, F along the x-axis at the end of the spring, the spring moves a distance x. The exprimental observation, due to Hooke (1676) is that the external force required to stretch a spring is given by,

$$F = kx \tag{3}$$

where k is the spring constant which has units (N/m) and x is displacement from the equilibrium position of the spring. If we compress or extend a spring which is initially at position 0, to a position x, we do work which we may calculate by using the formula,

$$W = PE_f - PE_i = \sum_i F_i \Delta x_i = \overline{F}x = \frac{1}{2}kx^2 \tag{4}$$

When we compress a spring, the work is converted into potential energy. If we suddenly release the spring, the potential energy is converted into kinetic energy. If we start with a spring at position x_i and compress it to position x_f, the change in potential energy is, $kx_f^2/2 - kx_i^2/2$.

Energy conservation

Work by an external force = Change in KE + Change in PE + Energy dissipated

Example. Consider a block which is placed against a spring which is compressed by a displacement $x = 1cm$. The block-spring system is placed at the bottom of a frictionless inclined plane. a) If the spring is released what is the maximum height up the inclined plane which the block will move? Take $k = 10^5 N/m$, $m = 1kg$. b) What is the speed of the block at height $h = 0.25m$? c) If the spring/block system is placed on a flat surface which has a kinetic friction coefficient of $\mu_k = 0.4$ what distance along the surface will the block move, after the spring is released?

Solution. a) No energy is dissipated. No work is done by an external force, so we can consider conservation of potential and kinetic energy. The initial energy is the potential energy of the spring. At the highest point the energy is just potential energy of the gravitational field. The maximum height is then found from,

$$\frac{1}{2}kx^2 = mgh_{max} \tag{5}$$

which $h_{max} = kx^2/(2mg) = 0.51m$.

b) At height 0.25, the block has a finite kinetic energy, so we have,

$$\frac{1}{2}kx^2 = mg(0.25) + \frac{1}{2}mv^2 \tag{6}$$

This implies that

$$10 - 2 * 0.25 * 9.81 = v^2 \tag{7}$$

From which we find $v = 2.26m/s$.

c) In this case all of the potential energy of the spring is lost due to friction. The work done against the friction force is $W = \mu_k mg\Delta x$, where Δx is the distance the block moves across the surface. This is set equal to the stored potential energy of the spring, so that

$$\mu_k mg\Delta x = \frac{1}{2}kx^2 \tag{8}$$

From this expression, we find that $\Delta x = 1.27m$.